Chapter 2

The Earth's Permanent Magnetic Field

The earth has a magnetic field, the earth's magnetic field. This field is not only attached to the earth, but it extends far outside of the atmosphere and has great importance for the movement of all charged particles and physical processes in near space. The earth's magnetic field is a part of her spacesuit, and protects us humans on earth against cosmic radiation, i.e., potentially damaging high-energy particles.

2.1 Introduction and history

The earth's magnetic field has been linked to navigation for over 1000 years. The compass, in different forms, was a valuable aid for ancient navigators, and it is not possible to imagine today's orienteering runners or hikers in fog and bad weather without it.

In the 15th century it became clear that the compass needle did not point exactly towards the geographic north pole. Navigators and cartographers learned to cope with this deviation, called declination (section 2.3). However, the declination is not the same everywhere. This variation became clear as early as in the 16th century, when the first measurements of declination where made. The first documented measurement of declination in Norway was made in 1596 by Willem Barentz.

The scientific exploration of the earth's magnetic field started with the Englishman William M. Gilbert's book, "De Magnete", published in 1600 (see figure 2.1). Gilbert made a magnetic model of the earth and showed by means of a compass that the direction of the magnetic field varied with where on earth one was located. He concluded,"Magnus magnes ipse est globus terrestris", which means, "The earth itself is a big magnet". In addition, he suggested that the source/reason for the magnetic field was to be found inside the earth. In the 18th century it became clear that the earth's magnetic field changes over time, with variations in both direction and amplitude over years, decades and centuries. Short-term variations where subsequently detected. Hans Christian Ørsted (1777-1851) discovered that electric currents cause magnetic fields. The pioneer in Norwegian science of the 19th century, Christopher Hansteen (1784-1873) got international attention because of his studies of the earth's magnetism.

Systematic observations of the earth's magnetic field started around 1840, about the same time as C.F. Gauss (figure 2.2) published the first realistic mathematical model to describe the geomagnetic field. During the first international polar year in 1882-83, attention was focused on obtaining international cooperation to map both place and time variations of the earth's magnetic field. Around 1850 it was shown that there was a connection between sunspot activity and perturbations in the magnetic field.
The Norwegian Kristian Birkeland (figure 2.3) organized major observation campaigns between 1896 and 1903) to map the impact of solar activity on the northern lights and on the earth's magnetism. Characteristic perturbations, so-called polar elementary magnetic storms, where observed (see section 2.5.3). Such storms are recurrent often after 27, 54 and 81 days; which means one, two or three sun rotation periods (see figure 2.14).
To explain these recurring magnetic storms, Birkeland suggested that confined areas on the sun send out a storm of charged particles (see chapter 6), and every time the active areas point towards the earth we experience magnetic storms and northern lights (see as well chapter 11).

2.1.1 Particle movement in the magnetic field

The earth's magnetic field dominates the movement of electrically charged particles in close space (chapter 10) and is of determining importance for the physics of the upper atmosphere. The field gives us some protection against particle radiation. Particle radiation from the sun consists of charged particles (electrons, protons and α-particles) streaming towards the earth with high velocity (chapter 6). They are deflected by the earth's magnetic field. This shielding is not perfect, because of the shape of the magnetic field. Particles

Figure 2.1: ***William M. Gilbert (1544 – 1603)***
Gilbert started the exploration of the earth's magnetic field. He demonstrated in his book "De Magnete" from 1600 (it has been translated and published in English in 1900) in a convincing way that the earth is a huge magnet and that the magnetic poles deviate quite a lot from the geographic poles. The front page of the second edition of the book is shown to the right

can penetrate into the atmosphere, especially in the earth's polar regions. If the particles move along the magnetic field lines, no force acts on them. This particle flow into the earth's atmosphere along magnetic field lines is the same way as particles get away from the sun's surface (corona holes); they stream out along open magnetic field lines.
The physics is as follows: Charged particles which move in a magnetic field are exposed to a force (Lorentz force) given by

$$\vec{F} = e\left(\vec{v} \times \vec{B}\right) \tag{2.1}$$

Here, $\vec{B}$ stands for the earth's magnetic field, e is the particle charge and v is the velocity. The magnitude of this force becomes:

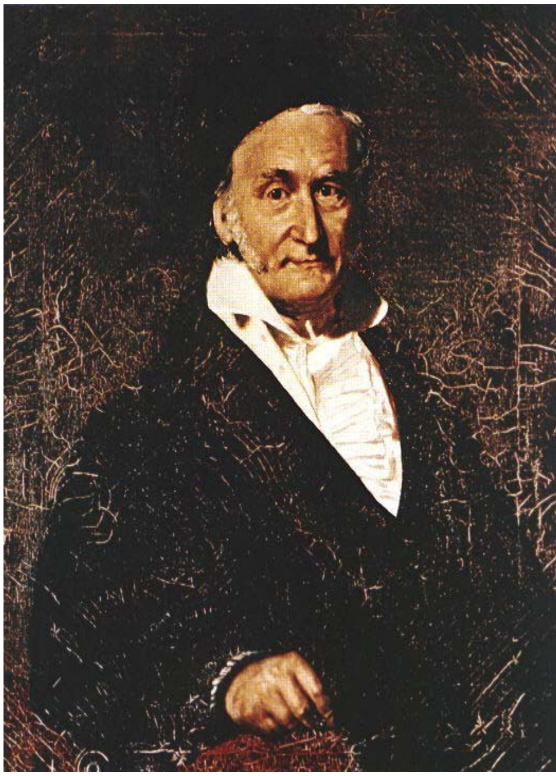
$$F = evB\sin\alpha \tag{2.2}$$

The angle between the particle's path and the magnetic field is the pitch angle α. For particles moving along the magnetic field lines, $\alpha = 0$, and so the force is zero.

Particle precipitation causes northern lights (aurora borealis) and southern lights (aurora australis), also called collectively polar light, as well as magnetic perturbation and electric currents.

2.1.2 The magnetic poles

July 1st 1831 was the "birthday" of the magnetic north pole. It was James Clark Ross on the Victory expedition who was the first man to reach the north magnetic pole. The pole was specified to 70°5'17"N and 96°46'45"V.



Carl Friedrich Gauss (1777 – 1855)
Figure 2.2: Gauss was a professor and director of the magnetic observatory in Goettingen, Germany. He worked on new theories which have been of fundamental importance for later geomagnetic research. Further on, he constructed new instruments for earth magnetic observations.

Kristian O. Birkeland (1867 – 1917)
Figure 2.3: Birkeland became a professor of physics at the age of 31. He was incredibly gifted and had an early urge for research. He started out as a mathematician, converted to theoretical physics and concentrated later only on experimental physics. Birkeland lead the geomagnetic and northern light research towards new dimensions. His activities had a complexity and profundity that was completely unknown in Norwegian research up to then. His work laid the fundament for Norsk Hydro.

The major goal of the Gjøa expedition (1903-1906), led by Roald Amundsen, was to locate the magnetic north pole. The Gjøa expedition magnetic instruments (one of these is shown on figure 2.5) had far better precision than the ones used by Ross on the Victory expedition. The expedition executed measurements south of King William's Land in Gjøahavn and at the King Point station (see table 2.1). They had continuous measurements for more than 700 days, and also made several sled expeditions to the area around the pole to measure the magnetic field. Based on some superficial observations we can make a table (table 2.1), which gives the coordinates for the magnetic north pole as Amundsen located it in 1904.

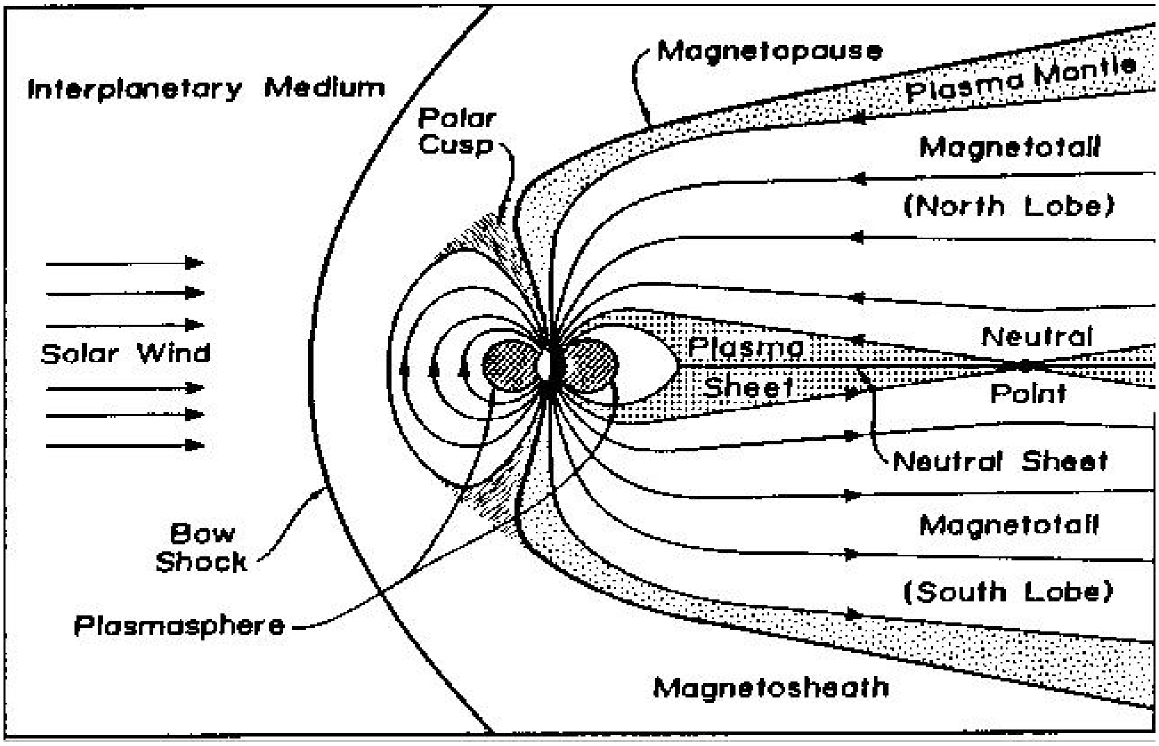


Figure 2.4: The figure shows the earth's magnetic field in the noon – midnight – plane (the sun is situated to the left). The shock front (bow – shock) arises when the supersonic solar wind interacts with the earth's magnetic field. The solar wind causes the earth's magnetic field to take the shape show in the figure (see chapter 9). The earth's magnetic field can lock up charged particles in a way that we get radiation belts. This is also called Van Allen belts, and they where discovered in 1958 by the means of a Geiger counter on board of the American research satellite Explorer I.

These observations are relatively consistent, but there is some uncertainty about how local anomalies affect the result.

From about 1910 on, several expeditions to the magnetic poles were made, and from 1948 on, the magnetic poles have been systematically surveyed. We have accurate measurements of how the field varies with time, and how the magnetic poles move. Right now, the pole in the northern hemisphere drifts in a north-west direction with a northerly speed of 24 km per year and a westerly speed of about 5 km per year. Its position in 1988 was at King Christian Island, with coordinates 75.3°N and 101.8°V. The magnetic poles have moved about 900 km during the 150 years that have passed since James Clark Ross found the pole in 1831.

Basis for determination	position north	position west
Declination measurement	$70^\circ 35'$	$96^\circ 10'$
Measurement of the horizontal intensity	$70^\circ 40'$	$96^\circ 05'$
Inclination measurement	$70^\circ 40'$	$96^\circ 55'$

Table 2.1 Determination of the magnetic north pole from Roald Amundsen's data

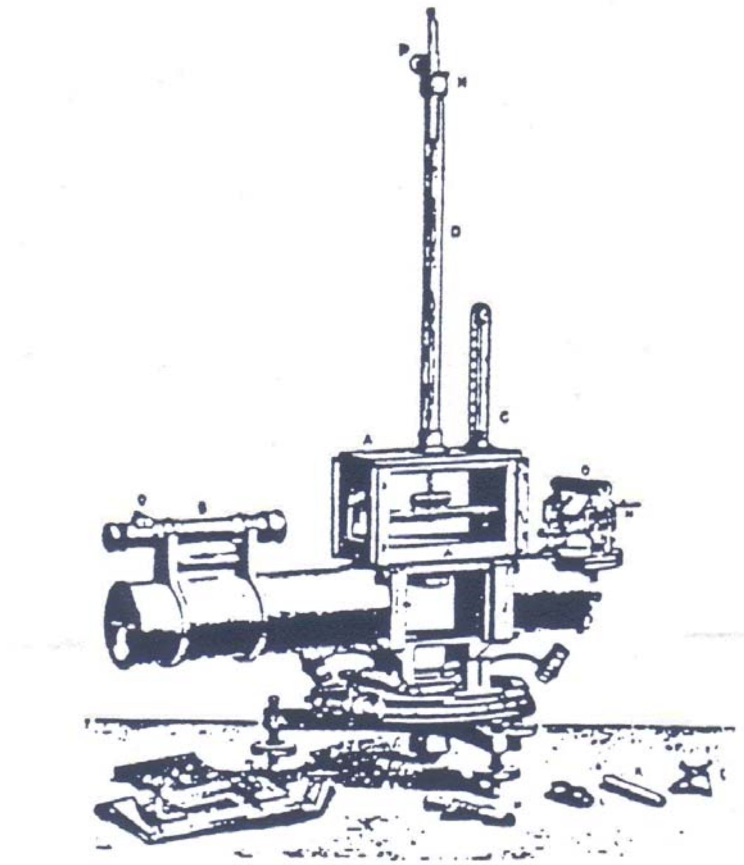


Figure 2.5: One of the instruments used by the Amundsen – expedition to find the magnetic pole.

2.2 Variations in the earth's magnetic field

The earth's magnetic field is not stable. Changes happen incessantly. Among others, the magnetic poles move from year to year.

Parallel with the studies of the magnetic field in space, new techniques for collecting and interpreting archeological data (burnt clay) and geological data (magnetic matter in volcanic mountains) have given new information about the earth's crust and interior, such as a better understanding of the inner electric currents (liquid iron and nickel inside the earth), which are main sources to the earth's magnetic field. Currents inside the earth vary slowly; over decades, centuries, or even longer time periods. (See figure 2.6, which illustrates some of these variations.)

Seismological measurements have shown that the earth's firm matter becomes liquid at a depth of about 3000 km. The temperature increases quickly towards the center of the earth. At temperatures above the Curie point (ca.1000K), matter looses its ferromagnetic properties and can no longer retain magnetization. Details about how the magnetic field develops and is maintained are still unknown. The most prevalent theory is the so-called self-exiting dynamo theory.

It is clear that the magnetic field has switched orientation many times in the past 4-5million years, which means that the compass needle has turned 180°. Existing measurements indicate that a polarity shift

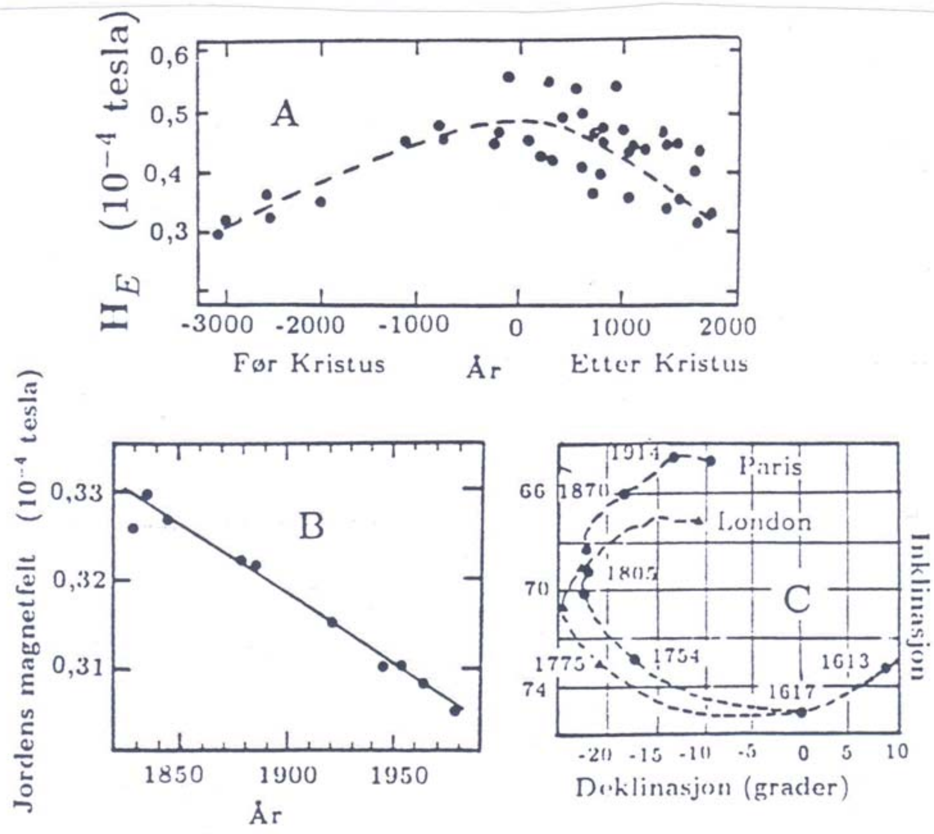


Figure 2.6: The magnetic field over the last 5 000 years (A) and 150 years (B). The field close to the equator (A) is determined from measurements of archeological samples, especially burned clay. In B, the field is determined by direct registration. The field decreases with around 10 nT per year. Variations in the direction of the magnetic field (declination and inclination) for London and Paris are shown in C. The accuracy in the observations is good from the middle of the last century. As shown here, the declination has changed with over 30° during this period.

happens such that the field decreases slowly towards zero, and then rebuilds with opposite polarity. But we have no satisfying explanation for this polarity shift.

It is reasonable to believe that high-energy cosmic radiation on the earth's surface will increase in periods with weak magnetic field, which may have an effect on life on earth. There are some indices that suggest that every time the magnetic field has changed orientation, there has been a climate change.
Changes in the earth's magnetic field during the last 5000 years, together with more detailed measurements for the past 150 years, are shown in figure 2.6 (A and B). We see that (in B) the field decreases by about 10nT per year. If the decrease continues like that, the field is going to be zero in about 2000-3000 years.

The average value of the earth's magnetic field changes from year to year (secular variations). The intensity decreases and Scandinavia's direction (declination) gets less western every year. In Tromsø, for example, the field decreases by 0.1%, and the direction turns by several arc minutes west each year. The changes are not constant with time. The intensity of the field has decreased much faster during the last 30-40 years. If the decrease continues like that, the field will be zero in only 1000 years.

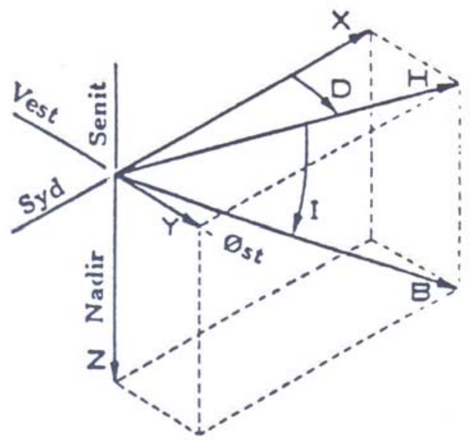


Figure 2.7: The figure illustrates the earth's magnetic field on the northern hemisphere and its different elements. B is the total magnetic field, H is the horizontal component, X is the component in the northern direction, Y in eastern direction and Z is the vertical component. D is the declination, I inclination. Notice that the declination (D) is positive east of geographic north and the vertical component (Z) is positive downwards. The horizontal component is defined as (..)

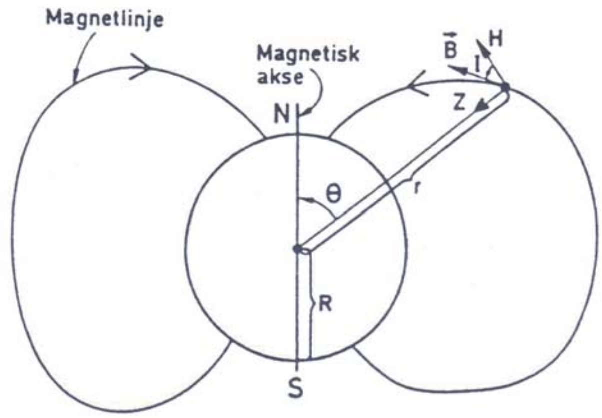


Figure 2.8: The simple model of the earth's magnetic field assumes a magnetic dipole located in the center of the earth. SN is the dipole axis, and where it intersects with the surface of the earth we have the dipole poles. Up to a few earth radii it is a good approximation to describe the earth's magnetic field by this dipole model. Some of the magnetic elements are as well marked in the figure. The angle θ between the observation location and the pole is the co latitude. The $\vec{B}$ *-field has the direction as indicated in the figure.*

2.3 The magnetic elements

At any arbitrary point, the geomagnetic field $\vec{B}$ is specified by direction and field strength. The vector can be specified by two angles together with the field strength, or by the value of three components that are perpendicular to each other. Because $\vec{B}$ is positive towards the earth on the northern hemisphere, the magnetic south pole is located on the northern hemisphere.

The earth's magnetic field is decomposed for a place on earth in the northern hemisphere in figure 2.7. We follow the international conventions where the *X* axis indicates north and the *Y* axis south. Then, the *Z* axis will be targeted at the center of the earth (nadir), and the negative *Z* axis towards zenith on the northern hemisphere.

The magnetic field has a horizontal component (*H*) and a vertical component (*Z*). We further on define two direction angles, inclination (*I*) and declination (*D*). Inclination is defined as the angle between the horizontal plane and the direction of the magnetic field. *D* and *I* are measured in degrees or radians. Declination is positive east of geographic north, whereas the vertical component (*Z*) is positive downwards. The set *B, H, X, Y, Z, D* and *I* is called the magnetic elements. The vertical plane through *H* is called the magnetic meridian plane. The earth's magnetic field is often described by the components *H, D* and *Z* or by *X, Y* and *Z*. If ,for example, *H* and *I* are known, we can find *Z* from the equation $Z = H \times \tan I$. See table 2.2 for the average value in Tromsø.

Magnetic field measured in Tromsø

	1970	1990
H Z B	≈ 11 200 nT ≈ 51 500 nT ≈ 52 660 nT	≈ 11 140 nT ≈ 51 600 nT ≈ 52 790 nT
I	78°	79°

Table 2.2: Average values for the magnetic field in Tromsø in 1970 and -90.

2.4 Theoretical model of the earth's magnetic field

It is practically impossible (and not necessary, either) to perform magnetic measurements for all places on the earth's surface. One tries, instead, to make a magnetic map of the earth's magnetism by means of mathematical models and existing observations. Around 1980 there were 250 observatories in continuous operation, seven of them in Norway. The best known, from which the data in table 2.2 is taken, is in Tromsø.
We describe the earth's magnetic field by means of a model, assuming that the field close to the earth's surface can be described approximately as a dipole field in the earth's center, or a homogenous magnetized globe in the dipole axis direction. This model is shown in figure 2.9. The unit for the magnetic intensity or the magnetic flux density is Tesla (*T*) or Weber per square meter [Wb/m^2]. *Wb*=1Volt second. Since Tesla is a large unit, we often use nano Tesla ($nT = 10^{-9}T$). In the cgs system, the unit Gauss is used (G): $1G=10^{-4}T$.
One imagines a magnetic dipole (bar magnet) inside the earth. The intersection points with the earth's surface define the magnetic poles, north and south.

We have as well other types of poles, the so-called *dip-poles* which are different from the axis poles. The dip-poles are the points on the earth's surface where the magnetic field is vertical (i.e., $H=0$, $B=Z$ and $I=90°$).

Since the earth is shaped like a globe, calculations get a bit more simple using polar coordinates. In polar coordinates (r and θ), where *r* is the radial distance from the center of the earth, and θ is the angle distance to the dipole axis (called co latitude), the magnetic potential becomes (with the dipole model) (see figure 2.8)

$$V_M(r,\theta) = -\frac{\mu_0}{4\pi r^3}\vec{M}\cdot\vec{r} = -\frac{\mu_0}{4\pi r^2}M\cos\theta \qquad (2.3)$$

where *M* is the earth dipole moment (ca. $8\cdot 10^{22} A\cdot m^2$), and μ_0 is the permeability in the unobstructed space. The potential V_M is composed of two terms; V_M^{inner} and V_M^{outer} . Since V_M^{outer} is caused by currents outside the earth, which on average contribute only about 1-2% to the permanent field, we will ignore it in the following calculations.

The magnetic field varies from place to place on earth because the angle θ varies. The field strength corresponding to this potential has the following horizontal and vertical components (refer to figures 2.7 and 2.8),

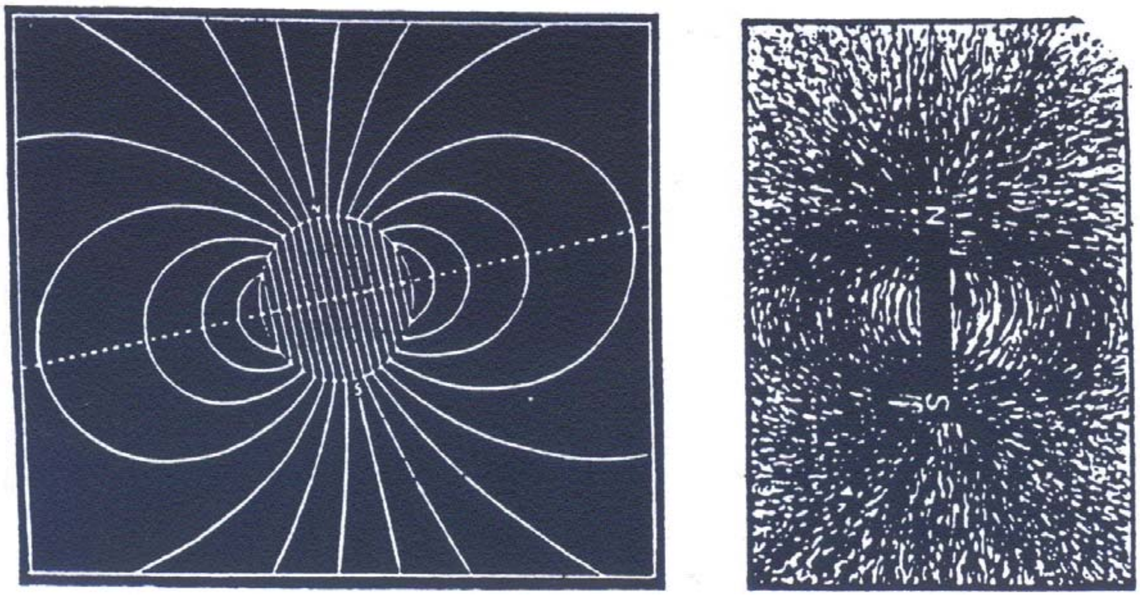


Figure 2.9: Models of the earth's magnetic field: To the left, the field of a homogeny magnetized sphere with open and closed magnetic field lines. It was this model of the earth's magnetic field that was used until around 1960. On the right part of the figure is shown how iron splinters on a glass sheet will form a pattern around a bar magnet. This pattern reminds of the one we see on the figure to the left.

$$
\begin{aligned}
H &= \frac{1}{r}\frac{dV_M}{d\theta} = \frac{\mu_0}{4\pi R^3} M \sin\theta = H_E\left(\frac{R}{r}\right)^3 \sin\theta \\
Z &= \frac{dV_M}{dr} = 2H_E\left(\frac{R}{r}\right)^3 \cos\theta \\
H_E &= = \frac{\mu_0}{4\pi}\left(\frac{1}{R^3}\right)M \\
\tan I &= \frac{Z}{H} = 2\cot g\theta
\end{aligned}
\tag{2.4}
$$

where H_E is the equator value ($Z=0$) of the horizontal component at the earth's surface (R is the earth's radius). The total field strength B is given by the following equation:

$$
B = \sqrt{\left(H^2 + Z^2\right)} = H_E\left(\frac{R}{r}\right)^3 \cdot \sqrt{\left(1 + 3\cos^2\theta\right)} \tag{2.5}
$$

At the poles, $\theta=0$, and at the equator $\theta=90°$. From equation 2.5 we get that if $\theta=\pi/2$ and $r=R$, $B=H_E$. Since we have a cosine term in equation 2.5, the earth's magnetic field (B) is twice as strong at the poles as at the equator. The field at the poles is ≈ *60 000nT*, whereas it is ≈ *30 000nT* at the equator. At the poles, the direction of the field is perpendicular to the earth's surface (figure 2.8), whereas at the equator, the field is parallel with the earth's surface. The direction of the field in space changes with magnetic latitude. Since the horizontal field is very weak north and south of 75° magnetic latitude, it is impossible to rely on a compass in the polar regions.

As long as we are located on the earth's surface, $r=R$ and the quantity $\left(\frac{R}{r}\right)^3$ equals 1. But if we travel away from the earth's surface, r increases, and the dipole field decreases. The reduction behaves like the third power of the distance; i.e., for $r=2R$ ($R \approx 6500km$) the field is just about 0.125 (12.5%) of the field at the earth's surface.

The equation for a magnetic force line in the meridian plane can be written

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{Z}{H} = 2\cot g\theta \text{ which has the solution } r = r_e \sin^2\theta \tag{2.6}$$

r_e is a constant which gives the distance from the center of the earth to the point where the field lines intersect the equator plane. The ending points (foot points) of the magnetic field lines are called magnetic conjugate points. In the polar regions, the magnetic field lines are open; i.e., just one end of the field line is connected to the earth.

2.4.1 Spherical harmonic analyses

To determine $\vec{B}$ at all places on the earth's surface, we have to carry out a harmonic analysis of the observed data. In a *spherical harmonic analysis*, the measured values are equalized. We get a pretty precise value for the earth's magnetic potential V_M, and we get distinct pole points for the field. The dip-dipoles (where $H=0$) prove to be located far (≥10°) from the dipole poles. This means that the dipole description of the field is not very good. The dipole description gives, for example, declinations for Scandinavia in the area 20°-30° west, whereas the observed values vary by some degrees east and west. From magnetic maps of the earth (figure 2.10) we see that the field is in fact not spherically symmetric; i.e., the dipole description is not good enough.

We are now going to describe the principles of a spherical harmonic analysis of the geomagnetic field. If the electric current between the earth and the earth's upper atmosphere is approximately equal to zero, we have that

$$\nabla \times \vec{B} = 0 \tag{2.7}$$

This means that there exists a potential so that $\vec{B} = -\nabla \cdot V_M$. Further on we always have that

$$\nabla \cdot \vec{B} = 0 \tag{2.8}$$

Combining equations 2.7 and 2.8 we get Laplace's equation:

$$\nabla^2 \cdot V_M = 0 \tag{2.9}$$

The scalar magnetic potential, caused by currents inside the earth, can be expressed in the following way,

$$V_M^{inside} = R\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n+1} P_n^m(\cos\theta); \quad \left[g_n^m \cdot \cos(m\lambda) + h_n^m \cdot \sin(m\lambda)\right] \tag{2.10}$$

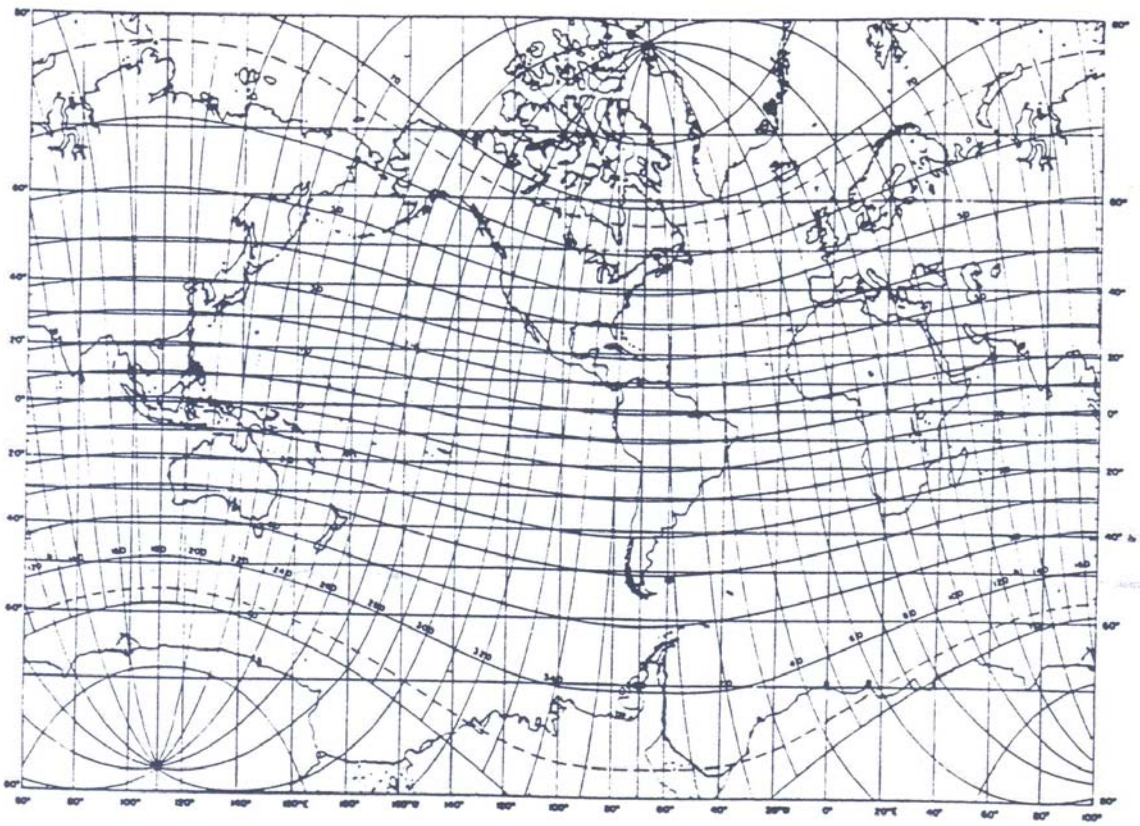


Figure 2.10: The earth's magnetic field after the IGRF (International Geomagnetic Reference Field) model from 1980. Areas with have the same magnetic field strength are conjunct by drawn through lines (isofield curves). We notice the peaks in north and south. The dipole poles and dip dipole poles are marked (N1,S1) and (N2,S2). There is a distinct minimum in the field outside Brasil (Atlantic Ocean minimum), an a peak over Siberia.

where *r, θ* and λ are the geographic sphere coordinates, radial distance, co latitude and eastern length.

P_n^m *cos θ* is associated and normalized Legrende polynomials. The coefficients g_n^m and h_n^m are called the Gaussian coefficients after C.F. Gauss (figure 2.2) who was a pioneer in spherical harmonic analysis of the earth's magnetism. In such an analysis, the measured data from all over the world is equalized. Equation (2.12) assumes that the sources of the $\vec{B}$ field are to be found inside the earth. By setting

$$X = \frac{1}{r} \cdot \frac{dV_M}{d\theta} \qquad Y = -\frac{1}{r\sin\theta} \cdot \frac{dV_M}{d\lambda} \qquad Z = \frac{dV_M}{dr} \tag{2.11}$$

we get the three components of the field expressed by g_n^m and h_n^m. How far we shall sum over n and m in (2.10) depends on the precision we demand. The first term in this sum (*n=1*) is the potential corresponding to a dipole inside the earth. There $P_0^1(x) = x$ and $P_1^1(x) = x'$ and we get the coefficients g_0^1, g_1^1 and h_1^1 Equation 2.10, when $P_0^1(x)\cos\theta = \cos\theta$ may then be written as

$$V_M^{inner} = R \cdot \left(\frac{R}{r}\right)^2 \cdot \left[g_0^1 \cos\theta + \left(g_1^1 \cos\lambda + h_1^1 \sin\lambda\right)\sin\theta\right] \tag{2.12}$$

where the first term is the dipole moment. The terms in front of g_1^0 become $R\times\left(\frac{R}{r}\right)^2\cos\theta$, which is the potential of a centered dipole with moment $M = g_0^1 \cdot R^3$, oriented along the earth's rotation axis. For the other two dipole terms (*cosλ·sinλ* and *sinλ·sinθ*), which lie in the earth's equator plane, we have $g_1^1 \cdot R^3$ and $h_1^1 \cdot R^3$. They point at 180°and 90° Ø geographical latitude. The resultant ($M=H_0 \cdot R^3$) of these three orthogonal dipoles has a magnitude determined by

$$H_0 = \sqrt{\left(g_0^1\right)^2 + \left(g_1^1\right)^2 + \left(h_1^1\right)^2} \text{ and co latitude by } \theta_0 = \cos^{-1}\left(\frac{g_1^0}{H_0}\right) \qquad (2.13)$$

2.4.2 Electric dipole description

To be able to get a good description of the magnetic field, we have to calculate higher order terms, i.e., quadruple and octuple terms, and so on. Further on, we have to consider the magnetic anomalies in the earth's surface, as well as include electric currents in the earth's upper atmosphere. By adding quadruple terms to the dipole terms we get what is called the **electric dipole determination**.

While the dipole terms decrease with distance from the earth surface as r^{-3}, the quadruple and octuple terms will decrease with r^{-4} and r^{-5}. Therefore, magnetic anomalies in the earth's surface contribute little in great distances (>*3R*) from the earth's surface. The eccentric dipole description is used a lot in detailed analysis of geographical data. It has the same direction and strength as the centered dipole, but its origin is displaced ≈ 400km from the center of the earth in the direction 10°N and 150°E.

By taking the sum up to *m=n=6,* which means taking in account 48 terms, the field on the earth's surface is described very well (error ≤ 1%). The observed field differs a lot from an eccentric dipole field in distances greater than four to five earth radii (see figure 2.4), but it still is approximately symmetric about the magnetic equator. The solar wind causes the deformation of the earth's magnetic field, especially in great distances from the earth.

The theory with the dipole centered in the inside of the earth gives a rough approximation to the real $\vec{B}$ - field . The deviation from the dipole symmetry can be illustrated by the value of the declination in Scandinavia. A big part of the uncertainty can be removed if we assume an eccentric dipole. We displace the center of the dipole 426km in the direction 15.6° N and 150.9°Ø. Further on we change the direction of the dipole in a way that it intersects the earth's surface at ca. 81.0°N, 275.3°Ø and at 75.0°S, 120.4°Ø. This helps a lot, but still there is deviation from the observed values. By series expansion of the magnetic potential, and determining the coefficients by the least square method, one can achieve as good an approximation as one wishes, if one just takes in account enough terms.

The clear maximum peaks in the earth's magnetic field that we find in Siberia (see figure 2.10) were first explored by the Norwegian, Christopher Hansteen, from 1828 to 1830. He established several measuring stations, and had an arrangement with some Norwegian seamen for them to observe the magnetic field on their journeys. On the basis of a huge amount of data, he drew the first realistic map of the earth's magnetic field (the same type as shown in figure 2.10). Hansteen developed, as well, a theoretical model for the earth's magnetic field.

Hansteen was a pioneer in Norwegian science. In 1814 he became lecturer in applied science at the university of Christiana, and two years later professor in astronomy and geophysics. His name is especially associated to the exploration of the earth's magnetic field and the Northern light. Hansteen performed several expeditions. The most known one is a 25 month journey to Siberia to examine whether the earth has two or more magnetic axis poles. Hansteen is as well known for his work with the almanac and for a new, modern astronomic observatory which was built in Christiana around 1840.

Figure 2.11: ***Christoper Hansteen (1784 – 1873)***

2.4.3 Magnetic coordinates and time

The earth's magnetic field plays a dominant role for the physics of the upper atmosphere. It is therefore expedient to refer to a magnetic time-and coordinate system.

The simplest geomagnetic coordinate system builds on the dipole description. For dipole latitude and dipole length we use λ and φ, respectively. The co-latitude is then given by $\theta = 90° - \lambda$. Magnetic length is positive east of the meridian through the magnetic and geographical pole. We are going to symbolize geographical latitude (measured from the rotational pole) and length by b and l. The geographic coordinates for the dipole-pole A is b_0 and l_0, whereas this pole's co-latitude is $\theta_0 = 90° - b_0$. For the geographic coordinates (spherical geometric) we get the following expression for the magnetic latitude and length (λ and φ)

$$\cot g\lambda = \cos\phi \cdot \cot g(\beta - \theta_0) \tag{2.14}$$

$$\tan\phi = \frac{\tan(l - l_0)\cdot \sin\beta}{\sin(\beta - \theta_0)} \tag{2.15}$$

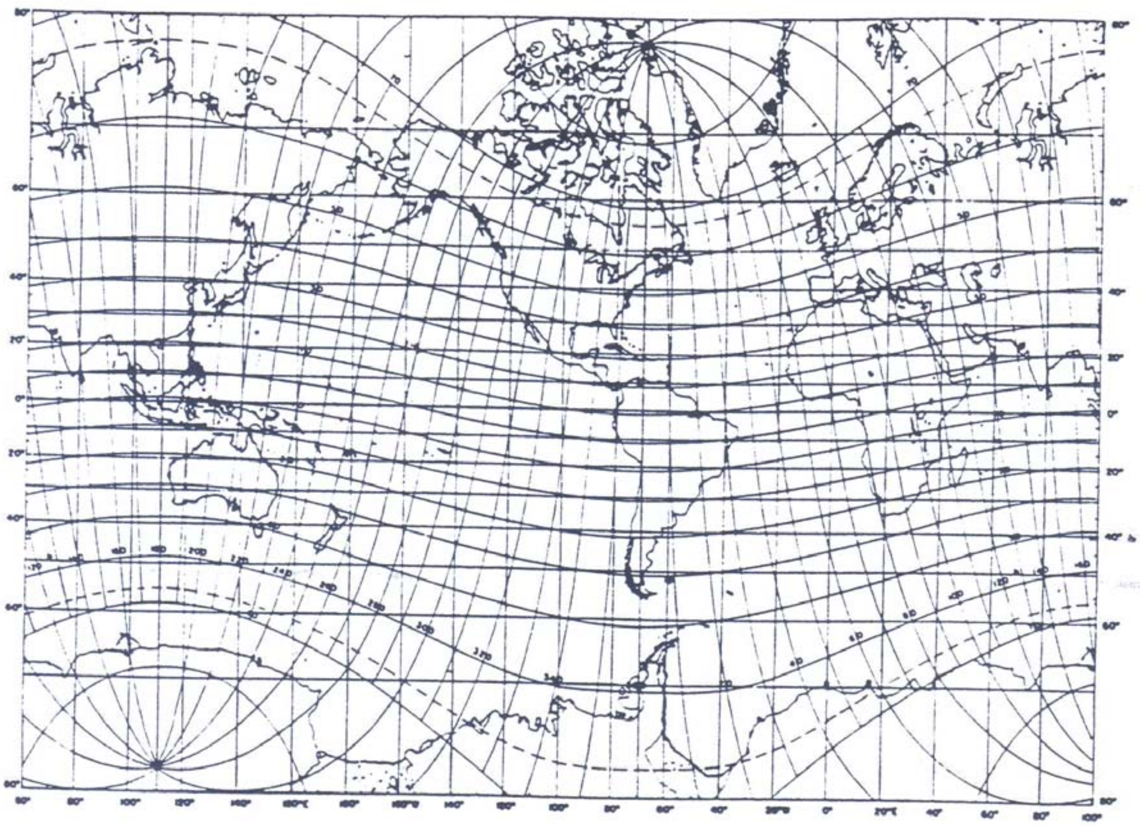


Figure 2.12: Mercator-projection of the earth overlaid with magnetic dipole coordinates

where *tan β = tan b·cos (l - l₀)* In case of*(l - l₀)=90°*, φ is not defined.

Figure (2.12) shows a map where both the geographical and the dipole system are shown. The sun's direction in relation to the magnetic coordinate system defines magnetic time in the same way as our local time is determined from a geographical coordinate system in relation to the sun. The geomagnetic time angle (φ) is the angle between the geomagnetic meridian plane, where the observatory is located, and the plane which is 180°displaced from the plane the sun is located in. A rotation of 15° corresponds to an hour as well in this coordinate system. The geomagnetic time t_M (in degrees) relative to the local time *t* for a point on earth (geographic length *l* and magnetic length φ) are given by

$$t_M = \phi + \arcsin\left[\frac{-\cos\delta \cdot \sin(l - t - l_0)}{\sqrt{l - [\cos\theta_0 \sin\delta - \sin\theta_0 \cos\delta \cos(l - t - l_0)]^2}}\right] \tag{2.16}$$

where δ is the angle with the ecliptic (the sun's declination) (i.e., the angle between the direction of the sun and the earth's equatorial plan).

New observations, especially satellite observations, have shown that the simple dipole description does not represents the earth's magnetic field very well, especially at great distances from the earth. We return to this topic later on in section 2.7.

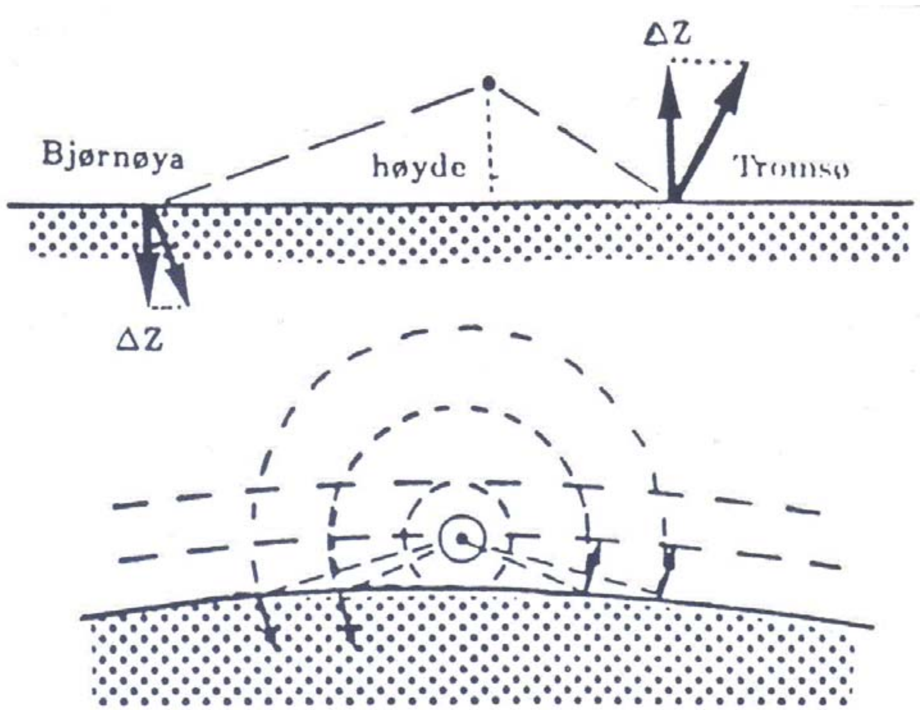


Figure 2.13: The upper part of the figure shows triangulation of the electric current system by means of the magnetic registrations from Bjørnøya and Tromsø. The lower part shows the magnetic power lines around a horizontal conductor with current direction out of the paper plane (marked by(...))

2.5 Magnetic variations

Magnetic variations in the earth's magnetic field can be divided into two main categories. The first are longtime variations ($10\text{-}10^6$ years), which have their cause in the interior of the earth (section 2.2). The second are variations due to electric currents in the upper atmosphere and/or in close space. This second type can last from seconds to one sunspot period.

Electric currents in the upper atmosphere and close space give the following three characteristic disturbances at the earth's surface:

- Daily variations due to neutral winds in the upper atmosphere.
- Disturbances in the polar regions - magnetic sub storms - mainly due to electrojet.
- Magnetic storms arising in near space after a fast increase of the particle population and energy.

We will take a closer look on these disturbances in the next sections.

2.5.1 Daily disturbances

The daily variations arise from movements in the ionosphere, especially in the E-layer (95-150km). The movements are caused by an irregular distribution of solar heating (a sort of tide effect), and they are greatest in summer. We call this effect S_q (*S* for sun and *q* for quiet). The S_q -effect is so small (typically $\approx 10nT$) that one normally does not take it into account. The hypothesis about the daily course in the magnetic field was first mentioned by Balfour Steward in 1884, but it was Kristian Birkeland who first pointed out that the main source for the magnetic disturbances are electric currents in the upper atmosphere.

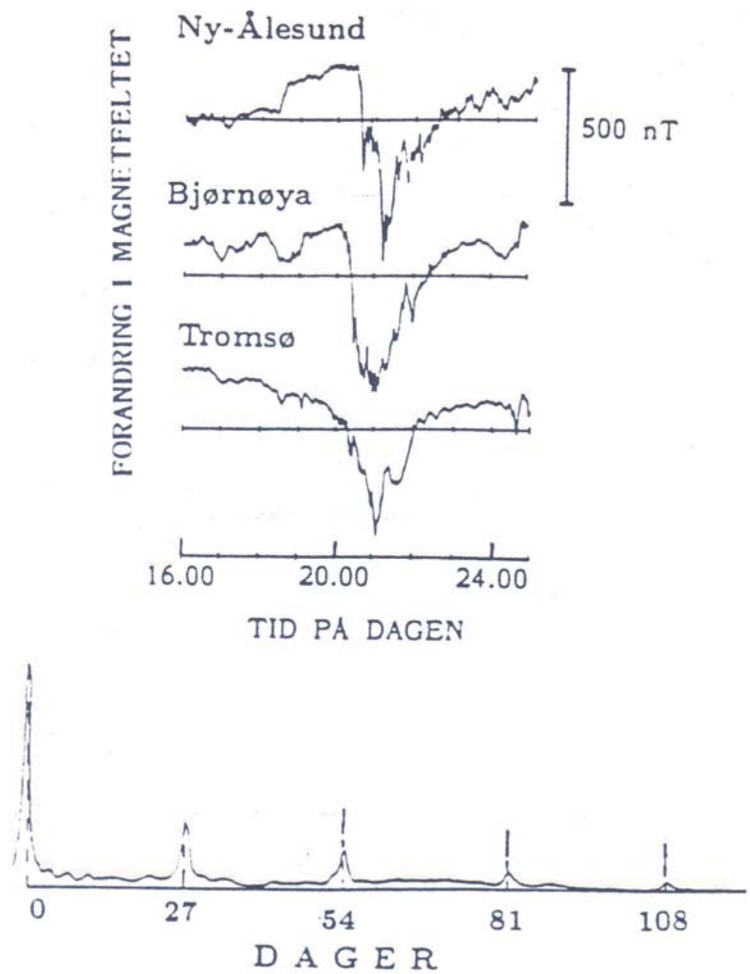


Figure 2.14: The upper part of the figure shows how a magnetic sub storm (September 19th,1977) lead to changes of the horizontal component of the earth's magnetic field in Ny Ålesund, at Bjørnøya and in Tromsø. The lower past of the figure shows how a magnetic sub storm can come back after one or more rotations of the sun.

There is another, minor, effect following the moon phases. A very detailed analysis of the magnetic data is necessary to determine the moon's contribution to the daily magnetic disturbances.

2.5.2 Disturbances in the Polar Regions

Disturbances concentrated in the polar regions (i.e., on the pole side of ≈ *60°* geomagnetic latitude) occur due to powerful electric currents in the highest regions ≈ *100-150km*.

In the polar regions, the ionization rate (i.e., the relationship between the number of ionized and neutral molecules/atoms) is especially high in the upper air layers, because there, the atmosphere is also bombarded with electrons and protons, which easily penetrate into these regions. This particle flow, which causes the northern and southern lights, creates powerful electric currents in this high region. It is therefore natural to expect that these currents will cause magnetic field disturbances on the earth's surface. The fact that magnetic field disturbances coincide with strong northern lights activity was proved more than 100 years ago. The electric current density is given by:

$$\vec{j} = ne(\vec{v}_i - \vec{v}_e) = \sigma\vec{E} \qquad (2.17)$$

where n is the density of free charges, e is the elementary charge, $\vec{v}_{i,e}$ is the velocity vector for ions and electrons, σ is the electric conductivity in the ionosphere, and $\vec{E}$ is the electric field at the point. The electric current flows, as Kristian Birkeland suggested almost 100 years ago, at an altitude between 100km and 150km ,in a region we now call the ionosphere. By integrating over the current density layer, we get the so-called height-integrated current ,

$$\vec{J} = \overleftrightarrow{\Sigma} \bullet \vec{E} \qquad (2.18)$$

where $\vec{E}$ is the electric field of the ionosphere, and $\overleftrightarrow{\Sigma}$ is the height-integrated conductivity tensor. We get the total current that flows in the atmosphere by integrating in an north-south direction, because the current flows in an east-west direction inside a magnetic latitude limited region, called the auroral oval. Typical values for the field ($\vec{E}$) and the height-integrated conductivity (Σ) during medium-to-strong northern light eruptions is *100mV/m* and *10Ω*, which gives a current of about *1A/m*. The total current strength over a 1000km-wide current zone can be around 1 million amperes. These currents result in powerful disturbances in the magnetic field measured at the ground, especially at night. The currents seem to be concentrated as an almost-line, forming a current element along one part of the northern light zone. Birkeland studied these simpler currents by triangulating from several observatories, after a principle sketched in figure 2.13. The currents are situated at 100km-150km altitude, with a amperage on the order of 10^6 - $10^7 A$.

Examples showing the variation of the horizontal component of the earth's magnetic field at three Norwegian observatories are shown in figure 2.14. The disturbances can reach up to *100-150nT*, which is about 2% of the total field at the location, but that is seldom. The duration of such magnetic sub-storms is typically some ten minutes, up to a couple of hours. In Tromsø, which is centrally situated in the auroral zone, one or more such sub-storms are registered every evening/night. In the Oslo area, the incidence is less frequent. Sub-storms are regional, and occur almost always inside the auroral zone.

2.5.3 Magnetic storms

Powerful magnetic storms, global in spatial reach, occur from time to time. They appear to be due to electric storms in our near space, especially in a ring-shaped electric current locted close to the equatorial plane at 4-7 earth radii distance. The effect of this ring current on the ground is therefore strongest at low latitudes; i.e., closer to the equator.

The duration of these storms can be everything from some minutes up to a maximum of two to three days. Singular powerful currents repeat after 27 and 54 days (see figure 2.14). This recurrence is why it is natural to assume that these currents are related to more active regions on the sun, because we know that the same regions point towards the earth with a 27 days recurrence period. The total energy being dissipated in a magnetic storm can be calculated to be up to a billion kilowatt hours ($10^9\,kWh \rightarrow 10^9\,PJ$).

2.6 Measurement instruments

Accurate and continuous measurements of the earth's magnetic field began around 1840. It was C.F. Gauss who was central in these measurements. A sketch of the equipment that was used for more than 100 years to measure the earth's magnetic field is shown in figure 2.15.
The core of the instrument is a permanent magnet (drawn like a small star in the figure), which is mounted in a way that it can rotate or swing free whenever the earth's magnetic field changes. A small mirror is mounted on the torsion radius. The mirror reflects a light ray and focuses it on rolling photographic paper. In addition, there is a steady mirror, which specifies the base line. Changes in the magnetic field leading the magnet and the mirror to turn are registered photographically by the light spot's movement on the paper. It

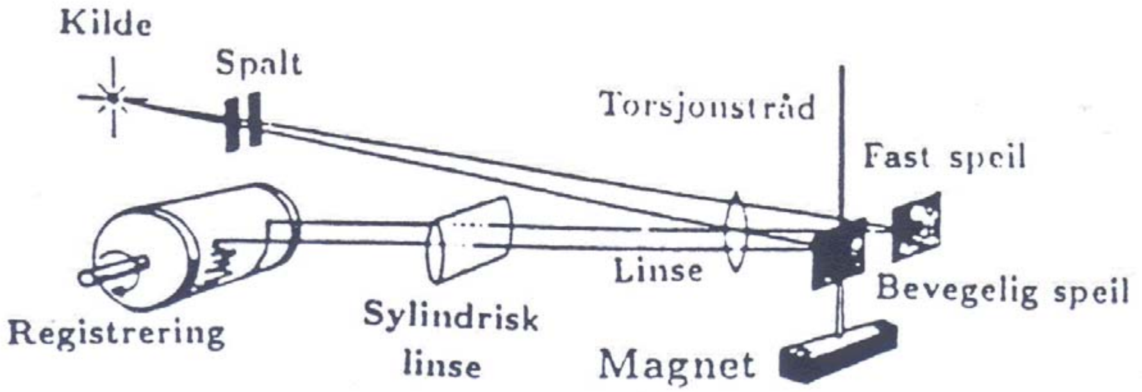


Figure 2.15: The figure shows the principle for how the earth's magnetic field is measured

is the deviation from the straight line that one is interested in (figure 2.14). (The deviation is compared with the northern light recording to examine the context between these phenomena.)

Today, digital manometers, developed in the past 30 years, are used for magnetic field recordings. They are based on other principles than the turning of a permanent magnet. For example, one type of instrument monitors the energy levels in rubidium atoms, which are influenced by magnetic field variations. Another technique, based on nuclear magnetic resonance (NMR), uses variations in the magnetic moment of protons, and is called proton resonance.. The instrument itself is known as a proton magnetometer.

2.6.1 Proton magnetometer

A proton placed in a magnetic field has energy E_{mag}, which we can write

$$E_{mag} = \mu \cdot B = g_N \cdot \beta_N \cdot I \cdot B \qquad (2.19)$$

Here, μ is the proton's magnetic moment, and B is the magnetic flux density (what we are going to measure). This can as well be written as $g_N \cdot \beta_N$ where g_N is a constant, which is characteristic for the nucleus. For a proton it is 5,586),β_N is the so-called nuclear magnetron (also a constant), and I is the proton's own spin (equals ½). Here we have the key to understanding the NMR method,because I can have two values; ± ½. We get two types of protons; those with spin +½ and those with spin – ½. Protons in a magnetic field will therefore split into two groups with different energy levels, depending on their nuclear spin value.
We can induce transition between these energy levels with the aid of radio waves. The condition for this transition to work is that the energy ($h\nu$) of the radio wave inducing the transition is precisely equal to the energy difference of the two proton groups. This resonance condition can be written as

$$h\nu = g_N \cdot \beta_N \cdot \mathrm{B} \qquad (2.20)$$

This gives us the flux density

$$B = \frac{h}{g_N \cdot \beta_N} \cdot \nu = C\nu \qquad (2.21)$$

where C is a constant ($=2.3488 \cdot 10^{-8} T/Hz$). For the earth's magnetic field with $50\,000nT$ (i.e., 0.5 Gauss), we get resonance for a frequency of $2.1287\ Hz$.

Today, modern electronics and computer techniques are used instead of photographic recording. Variations in the field can be written out in real time, and one can directly follow how, for example, the magnetic field varies during a northern light eruption.

Complete determinations of the earth's magnetic field are executed by the magnetic observatories around the world. The observatories in Norway at Bergen, Rørvik, Dombås, Andøya, Tromsø, Bjørnøya, Hopen and Ny-Ålesund survey the field continuously. At the northern light observatory in Tromsø, the magnetic field has been measured for more than 60 years. The fluxgate magnetometer is used at all stations in Norway. The observations are one part in the world-wide cooperation project and are among others used for studies of the magnetic disturbances in the Polar Regions and global magnetic storms.
Micro-pulsation instruments are used to map rapid magnetic field variations (< few seconds). The core of this instrument consists of big electric coils (with a diameter up to 1m, and with 10 000-20 000 copper wire whorls; often around soft metal cores), which are dug into the earth so that they lie steady. Variations in the earth's magnetic field result in electric currents in the coils. These currents can be intensified and recorded. Such induction - *search coil* - instruments are also used in rockets and satellites.

2.7 The magnetic field outside the earth

As shown in figure 2.4, the earth's magnetic field deviates a lot from a dipole field, especially in distances greater than four to five earth radii from the earth's surface. Interactions with the solar wind compresses the Earth magnetic field on the dayside and stretches it into a long tail on the night side Strong electric currents are induced (which again are surrounded by a magnetic field) at the magnetopause on the dayside, and far outside in the tail on the night side.

We will now calculate the stand-off distance to the solar wind in the equatorial plane on the dayside.
The solar wind presses on the magnetic field. In a certain distance r there will be a balance where the pressure from the solar wind equals the pressure from the earth's magnetic field. This distance (r) is defined as the range of near space. The mass of electrons and protons are m and M, the speed of the solar wind is v, and the particle density is n (there are as many electrons as protons). The kinetic pressure P_S from the solar wind is

$$P_s = n(n+m)v^2 \cong nMv^2 \tag{2.22}$$

We only consider the protons because their mass is 1836 times the mass of an electron. The counter-pressure P_B from the earth's magnetic field is

$$P_B = \frac{B_d^2}{2\mu_0} \tag{2.23}$$

B_d is the earth's magnetic field at the point we have balance, and μ_0 is the permeability for vacuum. The field B_d can be expressed by the field on the earth's surface, and we get from equation 2.4 for the magnetic field in the equatorial plane (θ=90°) that

$$B_d = H_E\left(\frac{R}{r}\right)^3 \tag{2.24}$$

Here, r is the distance we want to calculate. By setting in into equation 2.24 and setting P_S=P_B we get the following expression

$$nMv^2 = \frac{1}{2\mu_0}\left(\frac{R}{r}\right)^6 (H_E)^2 \tag{2.25}$$

This equation can be solved for *r*:

$$r = R \cdot \sqrt[6]{\frac{(H_E)^2}{2\mu_0 nMv^2}} \tag{2.26}$$

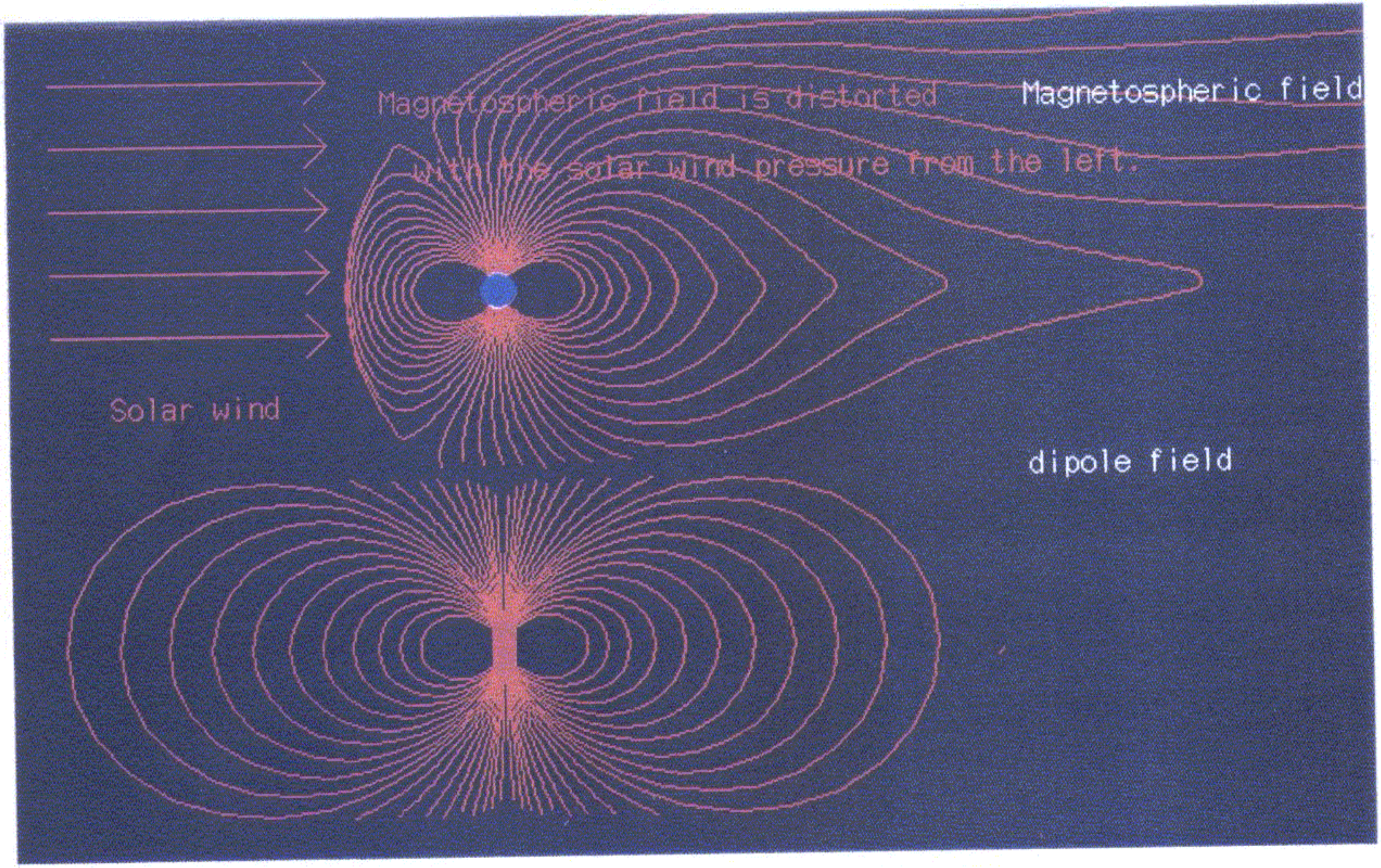
In this equation $n=8\cdot10^6 P/m^3$ and *v* is known from satellite observations. (The observed average value for *r* is *r=10.8R*; ≈*70 000km*). We then find that

$$r \cong 8 - 10R \tag{2.27}$$

if *v* varies from 300-500km /s. This means that the earth's magnetic field reaches out to about eight earth radii on the solar side. This simple calculation is not quite true because the solar wind will induce currents in the boundary layer at *r*. These additional currents mean that the field outside in space is much stronger than the original dipole field. When we take this into consideration, we find that the earth's magnetic field stretches out to about 11 earth radii during calm conditions on the sun. This distance will vary a lot (from 6 to 14*R*) depending on conditions in the solar wind. The greater the activity, the smaller *r* gets. The distance of the earth's magnetic field as a function of the activity on the sun has been precisely observed by satellites. The values calculated here fit well with the observed values. On the side of the earth turned towards the sun, the range of the earth's magnetic field is about 10 earth radii (*64 000km*). On the night side, on the other hand, the field stretches in a long tail (several hundred earth radii) because of the interaction with the solar wind (see figure 2.4)

For all distances other than towards the sun, the earth's magnetic field will extend more than 10 earth radii. Little is known yet about what the field looks like on the night side at great distances from the earth.

The two main models currently under discussion by scientists for the coupling between the earth's magnetic field and the magnetic field in the solar wind are the open and the closed magnetosphere. In the open model, the earth's magnetic field is directly joined with the $\vec{B}$ -field in the solar wind, whereas in the closed atmospheric model there is no contact between the earth and the solar wind's magnetic field.
By setting $P_s=P_b$ we ignore the solar wind's magnetic field, as we have discounted the contribution from the plasma in the magnetosphere. These modifications are not going to change much in the determination of *r*.


Magnetospheric field is distorted
with the solar wind pressure from the left.
Magnetospheric field
Solar wind
dipole field